EXERCISES.

251

LET $\Sigma_i p$ be the sum of any n ($<\frac{1}{2}m$) of the quantities $p_1, p_2, \ldots p_m$, taken at random; let $\Sigma_j p$ be a similar sum of n quantities not in $\Sigma_i p$; then will

$$\Pi \left\{ \frac{a_i + a_j}{2} \right\}^{(2n)!\,(m-2n)!} > \Pi a_i^{\,n!\,(m-n)!},$$

where Π denotes the product taken with regard to all the possible combinations of the quantities $p_1, \ldots p_m$ involved in a_i and $a_i + a_j$ respectively.

[*Ormond Stone.*]

252

$f(x)$ and $\varphi(e^x)$ being rational algebraic functions of x and e^x respectively, show that

$$\int f(x)\,\varphi\,(e^x)\,dx$$

depends upon integrals of the three forms,

$$\int \frac{du}{\log u}, \quad \int \theta^m \tan\theta d\theta, \quad \int \frac{du}{(u+c)^r \log u}.$$

[*R. A. Harris.*]

253

ANY curve not of degree $3m$, which is symmetrical with regard to an equilateral triangle, is circular. [*Frank Morley.*]

254

USING the complex variable, the points z_1, z_2, z_3 will form an equilateral triangle if

$$z_1 + \omega z_2 + \omega^2 z_3 = 0$$

where ω is an imaginary cube root of 1. Hence, show that

(a) If O is a point inside an equilateral triangle ABC at which the sides subtend angles α, β, γ, then the triangle whose sides are equal to OA, OB, OC will have angles equal to $\alpha - \frac{1}{3}\pi$, $\beta - \frac{1}{3}\pi$, $\gamma - \frac{1}{3}\pi$.

(b) If equilateral triangles are described on the sides of a plane polygon, all in the same sense, the centre of mean position of the corners of the triangles which do not coincide with those of the polygon, will coincide with that of the corners of the polygon. [*Frank Morley.*]

255

IN a regular polygon of m sides let r_1, r_2, etc., be the lengths of the lines from one corner to the others taken in order; then

$$1+\frac{r_{3r-m}}{r_p}=\frac{r_{2m-4p}}{r_{m-2p}},$$

where $p>\frac{1}{3}m$, $<\frac{1}{2}m$. [*Frank Morley.*]

256

PROVE the construction for the tangent at a cusp A of a quintic having cusps at A, B, C, D, E. Let BC, DE cut at P; let AQ be the fourth harmonic of AB, AC, AP in the involution defined by AB, AC and AD, AE; let AR be the ray conjugate to AQ; let AS be the fourth harmonic of AP, AQ, AR; let AT be the ray of the involution conjugate to AS; then AT is the required tangent. [*Frank Morley.*]

257

GIVEN three harmonic motions of equal periods, amplitudes a, b, c, and at right angles. Find the surface which includes every phase of the resulting curvilinear motion when the ratio of the rates of change of phase is given.

[*W. S. Franklin.*]

258

GIVEN a limited looped curve consisting of connected portions of known curves. Find two series of harmonic motions at right angles which, when compounded with a uniform motion of translation, will be equivalent to the motion of a point traveling along the curve with uniform velocity. [*W. S. Franklin.*]

259

FIND the general equations to the curve assumed by the water jet of a "Barker's Mill," the axis of rotation not being vertical. The velocity of issue, however, being assumed constant. [*W. S. Franklin.*]

260

A's coefficient of veracity is p, that of B is equally likely to be v or $1-v$. Required: (1) the probability of the truth of a statement to which they agree; (2) the probability of the truth of a statement which A makes and B denies.

[*Henry Heaton.*]

261

FIND the locus described by a point on the connecting rod of a steam engine when the piston rod produced does not cut the crank circle. [*T. U. Taylor.*]

262

DETERMINE the volume produced by revolving the curve $y^{\frac{1}{2}}-x^{\frac{1}{2}}=a^{\frac{1}{2}}$ about $x-a=0$. [*O. Root, Jr.*]

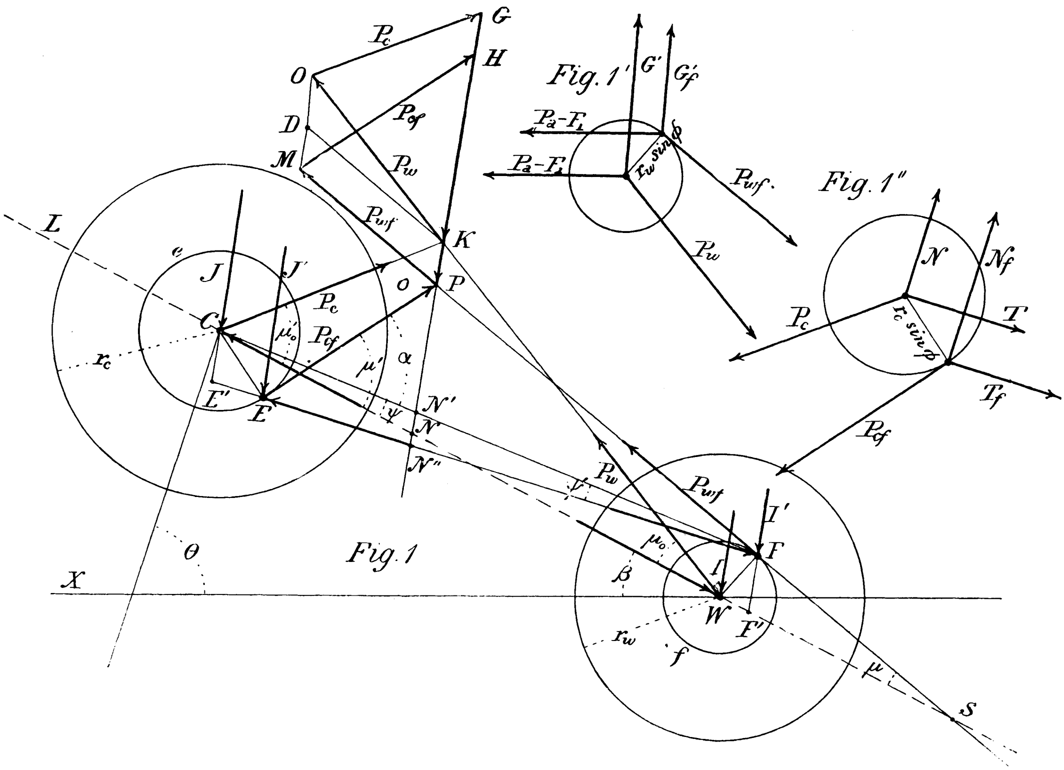
Fig. 1
Fig. 1'
Fig. 1''

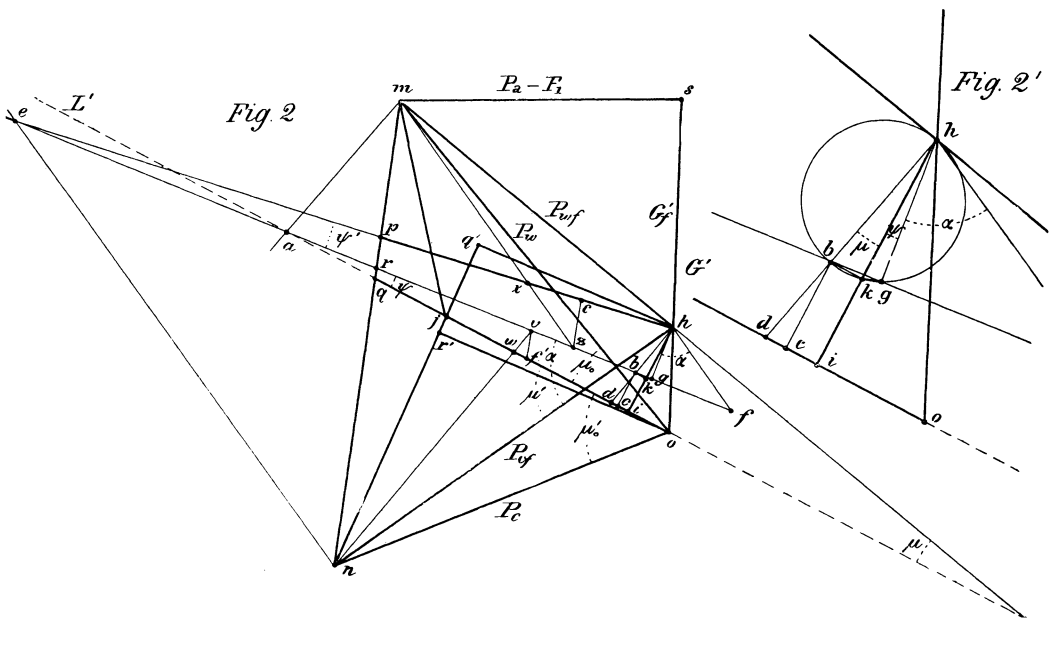
Fig. 2
Fig. 2'

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